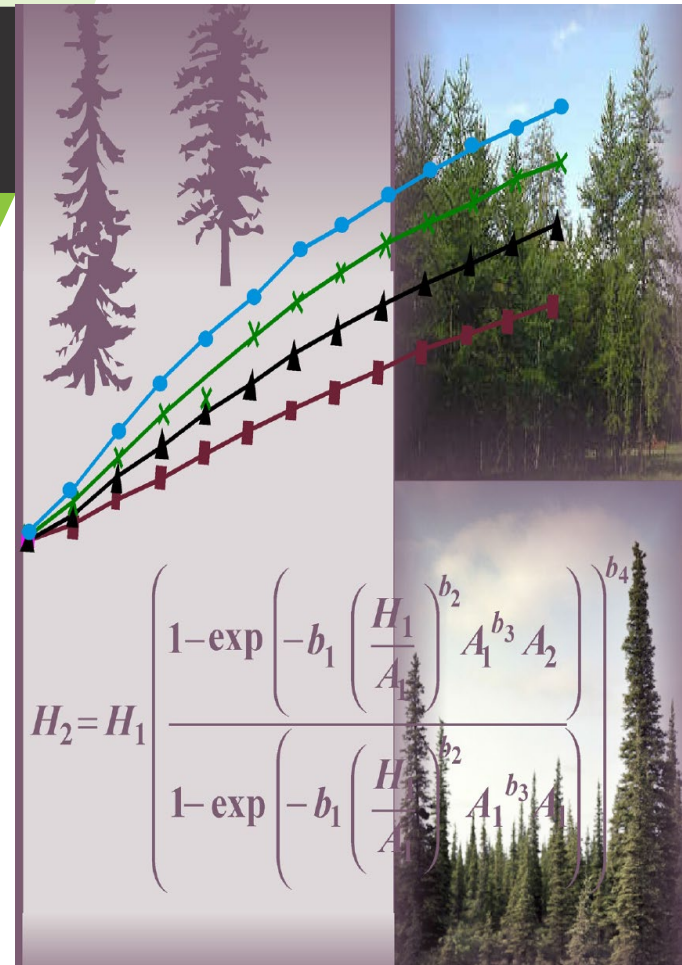# Developing Growth and Yield Models for White Pine and White Spruce Plantations

Ministry of Natural Resources and Forestry

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$$H_2 = H_1 \left(\frac{1 - \exp\left(-b_1 \left(\frac{H_1}{A_1}\right)^{b_2} A_1^{b_3} A_2\right)}{1 - \exp\left(-b_1 \left(\frac{H}{A}\right)^{b_2} A_1^{b_3} A_1\right)} \right)^{b_4}$$
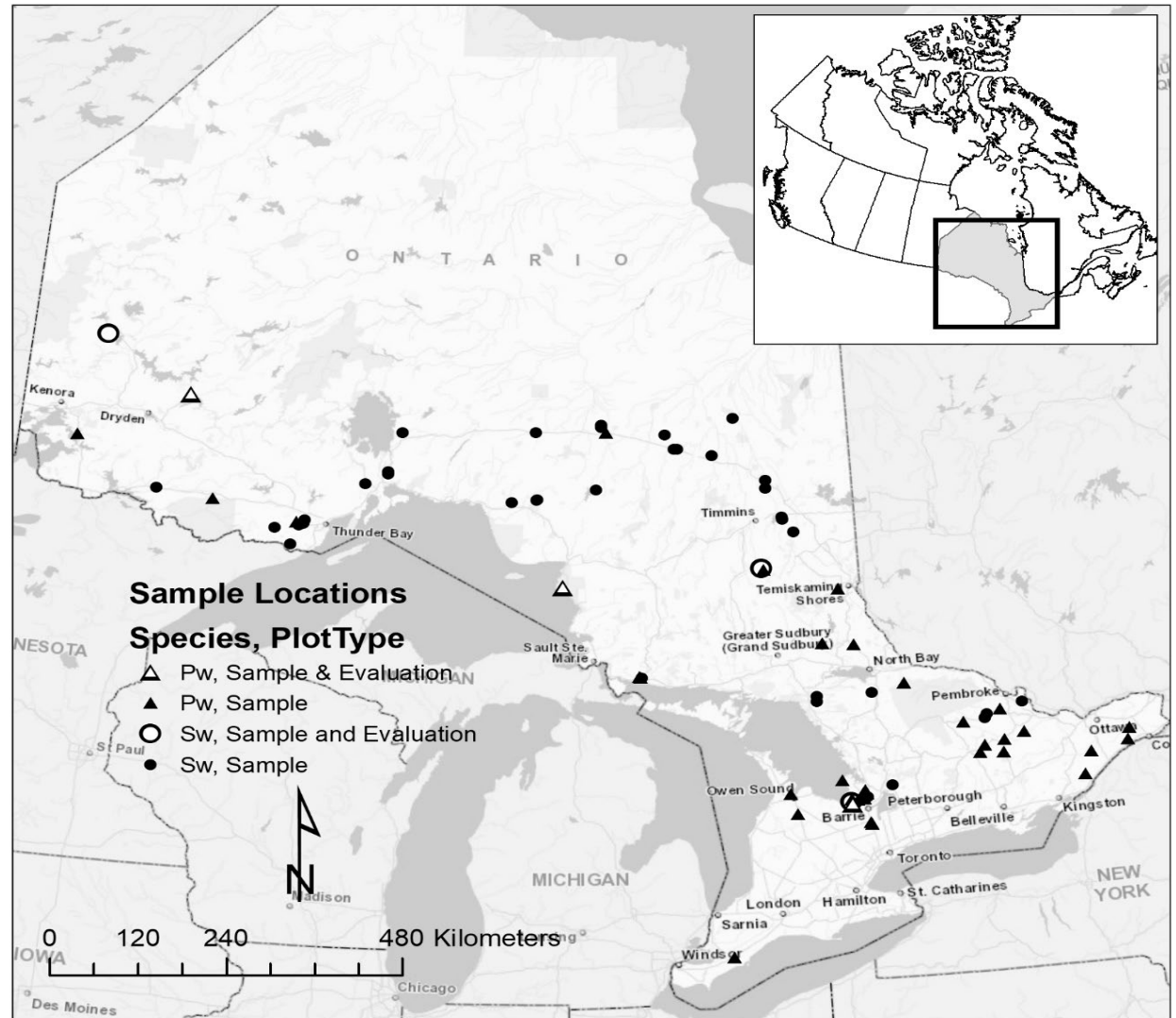
Objectives

Develop the following models for white pine and white spruce plantations

- inside and outside bark taper equations by incorporating stand density information

- merchantable and total inside and outside bark volume equations

- analyze climate effects on diameter growth and derive climate sensitive diameter growth equations

2

Ontario

Data

- 200 trees/species

- 40 sites/species

- 400 m^2 plot was established at each site that included at least 40 trees

- Plot size was increased if less than 40 trees

- Trees in the plot were divided into 5 BA classes

- 1 tree was randomly sampled from each basal area class

- 13 disks/tree

**Sample Locations
Species, PlotType**
△ Pw, Sample & Evaluation
▲ Pw, Sample
○ Sw, Sample and Evaluation
● Sw, Sample

Ontario

Data

Attribute	N	Mean	Std Dev	Minimum	Maximum
White spruce					
DBH (cm)	200	24.83	7.03	10.10	48.80
Height (m)	200	19.59	3.08	12.30	26.75
Crown ratio	200	0.453	0.127	0.181	0.804
BA (m^2 ha^{-1})	40	41.65	11.16	22.76	81.50
Trees ha^{-1}	40	1134.14	451.22	533.33	2625.00
QMD (cm)	40	22.46	4.70	14.90	38.97
White pine					
DBH (cm)	200	27.78	8.84	11.50	55.10
Height (m)	200	21.09	4.59	8.60	34.90
Crown ratio	200	0.409	0.132	0.139	0.913
BA (m^2 ha^{-1})	40	44.00	12.15	23.09	78.84
Trees ha^{-1}	40	975.08	451.72	366.67	2425.00
QMD (cm)	40	25.36	5.83	15.40	38.31

Ontario

Taper equations

Develop inside and outside bark taper equations for white spruce and white pine plantations by incorporating tree and stand characteristics
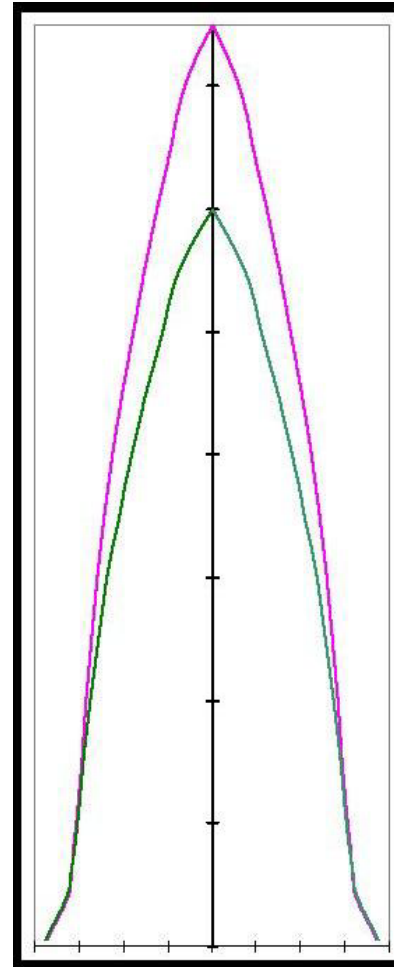
Same DBH

Same height

Ontario
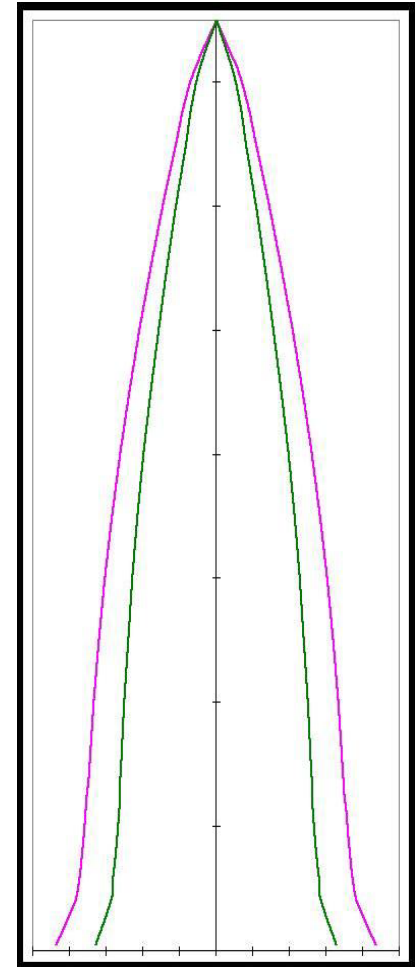
Taper equations

Sharma and Parton (2009)

$$\frac{d}{D} = \beta_0 \left(\frac{H-h}{H-h_D}\right) \left(\frac{h}{h_D}\right)^{\beta_1+\beta_2 x+\beta_3 x^2} + \varepsilon \qquad (1)$$

where

d = diameter inside or outside bark at any given height h (m),

D = diameter at breast height (DBH) outside bark (cm),

H = total tree height from ground to tip (m),

h_D = breast height (m),

$x = h/H$,

ε is the error term, and β_i (i = 0, 1, 2, and 3) are parameters.

Ontario

Taper equations

$$\frac{d}{D} = \beta_0 \left(\frac{H-h}{H-h_D}\right) \left(\frac{h}{h_D}\right)^{\beta_1 + \beta_2 x + \beta_3 x^2 + \beta_4 f(sd) + \beta_5 \, CR} + \varepsilon \qquad (2)$$

where, $f(sd)$ is a function of stand density, CR is the crown ratio and β_4 and β_5 are the parameters to be estimated.

White spruce

$$y_{ijk} = (\beta_0 + b_{0ij}) \left(\frac{H_{ij}-h_{ijk}}{H_{ij}-h_D}\right) \left(\frac{h_{ijk}}{h_D}\right)^{(\beta_1 + b_{1ij}) + (\beta_2 + b_{2ij})x_{ijk} + \beta_3 x_{ijk}{}^2 + \beta_4 \frac{QMD_i}{D_{ij}}} + \varepsilon_{ijk} \qquad (3)$$

White pine

$$y_{ijk} = (\beta_0 + b_{0ij}) \left(\frac{H_{ij}-h_{ijk}}{H_{ij}-h_D}\right) \left(\frac{h_{ijk}}{h_D}\right)^{(\beta_1 + b_{1ij}) + (\beta_2 + b_{2ij})x_{ijk} + (\beta_3 + b_{3ij})x_{ijk}{}^2 + \beta_4 \frac{QMD_i}{D_{ij}}} + \varepsilon_{ijk}$$

$$(4)$$

$$QMD = f\,(BA,\,TPH) = 112.8379 \sqrt{\frac{BA}{TPH}}$$

Ontario

Taper equations

Parameter estimates and fit statistics for Eq. (3) estimated using NLMIXED procedure in SAS for inside and outside bark diameters of white spruce trees grown in plantations across Ontario.
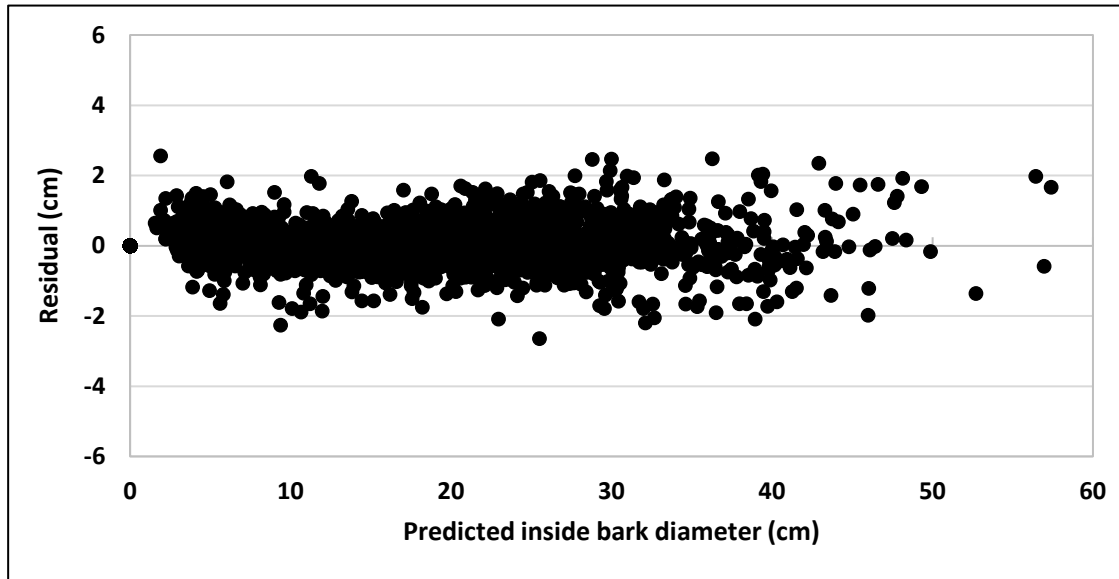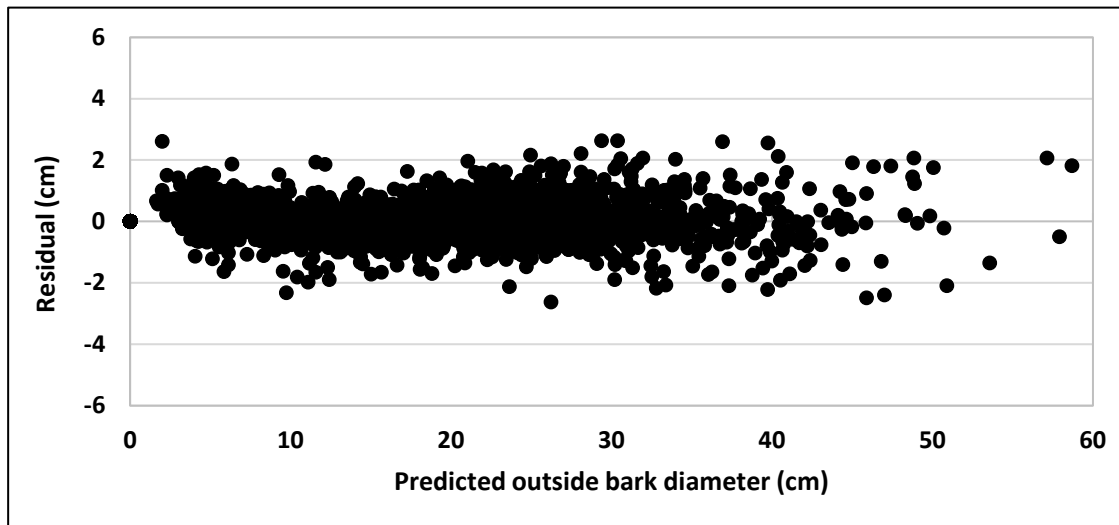
Parameters	Inside bark		Outside bark	
	Estimates	SE	Estimates	SE
β_0	0.98050	0.00147	1.00680	0.00127
β_1	-0.07500	0.00799	-0.07455	0.00777
β_2	0.43900	0.00772	0.43810	0.00763
β_3	-0.17080	0.00778	-0.15940	0.00773
β_4	0.02070	0.00816	0.01861	0.00793
σ^2	0.00055	0.00002	0.00058	0.00002
var (b_0)	0.00026	0.00004	0.00015	0.00003
var (b_1)	0.00070	0.00007	0.00066	0.00007
var (b_2)	0.00415	0.00048	0.00394	0.00046
AIC	-11591	--	-11544	--

Ontario

Taper equations

Parameter estimates and fit statistics for Eq. (4) estimated using NLMIXED procedure in SAS for inside and outside bark diameters of white pine trees grown in plantations across Ontario.

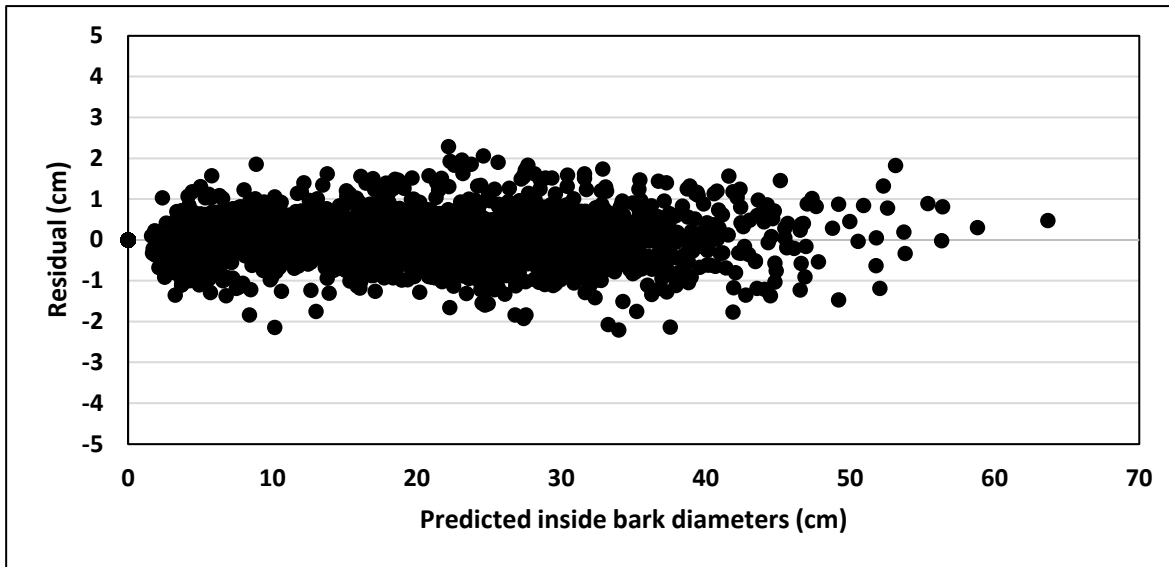| Parameters	Inside bark		Outside bark	
	Estimates	SE	Estimates	SE
β_0	0.93620	0.00143	0.99350	0.00116
β_1	-0.05525	0.00514	-0.06736	0.00512
β_2	0.48900	0.01292	0.49800	0.01210
β_3	-0.28370	0.01770	-0.26660	0.01667
β_4	0.02279	0.00509	0.02046	0.00509
σ^2	0.00045	0.00001	0.00051	0.00002
var (b_0)	0.00046	0.00006	0.00033	0.00003
var (b_1)	0.00031	0.00004	0.00031	0.00004
var (b_2)	0.02814	0.00356	0.02502	0.00298
var (b_3)	0.05423	0.00671	0.04744	0.00574
AIC	-11921	--	-11854	--

Ontario

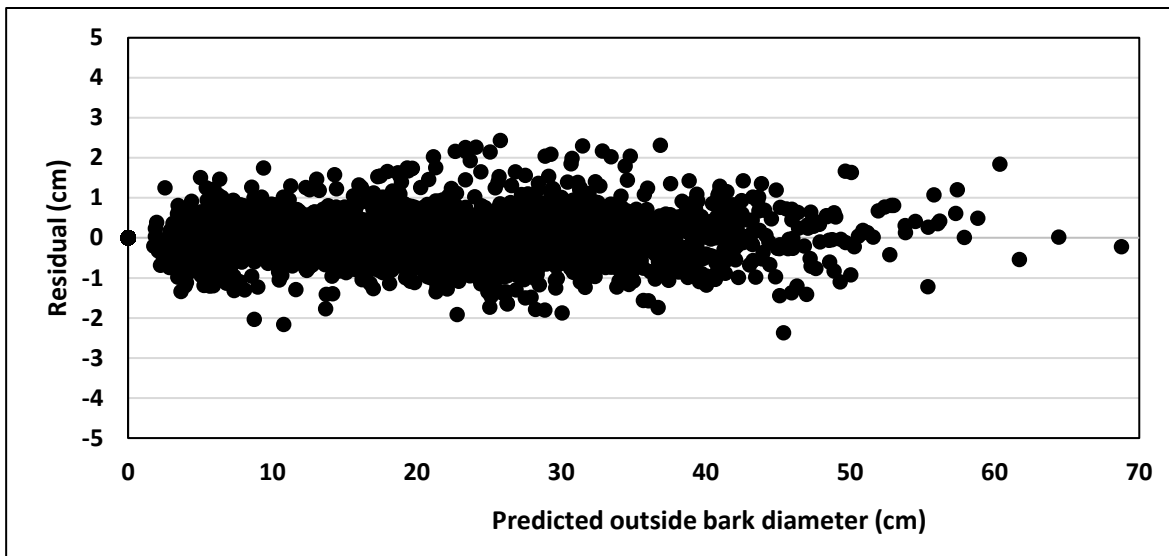# Taper equations

Residuals (observed – predicted) of white spruce plantation inside and outside bark diameters
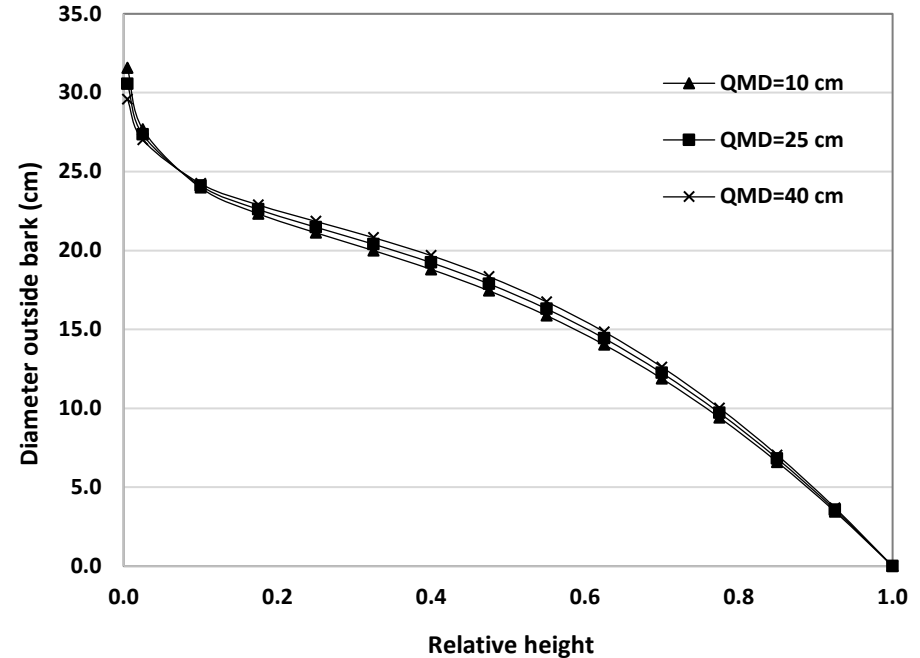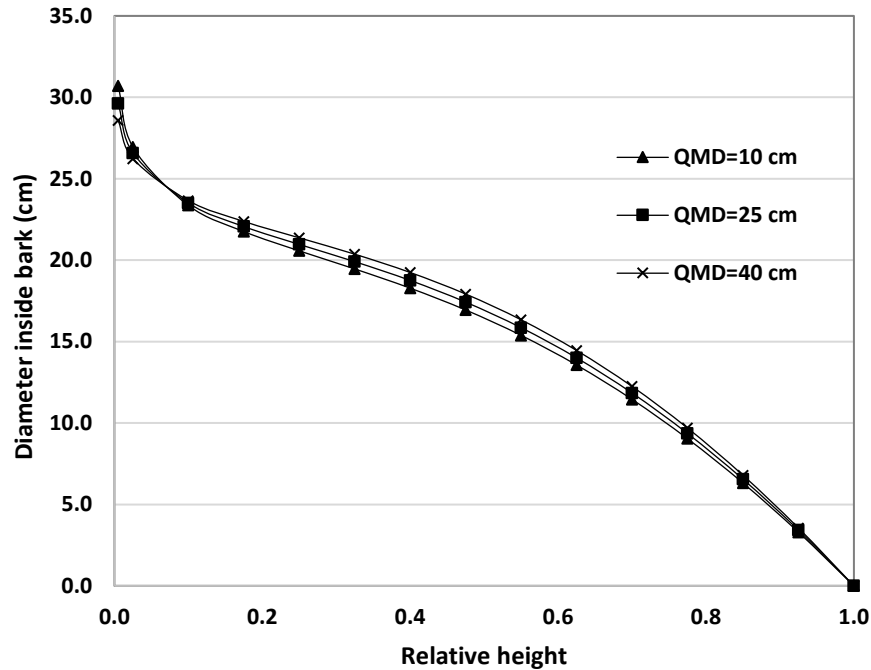
Ontario

Taper equations

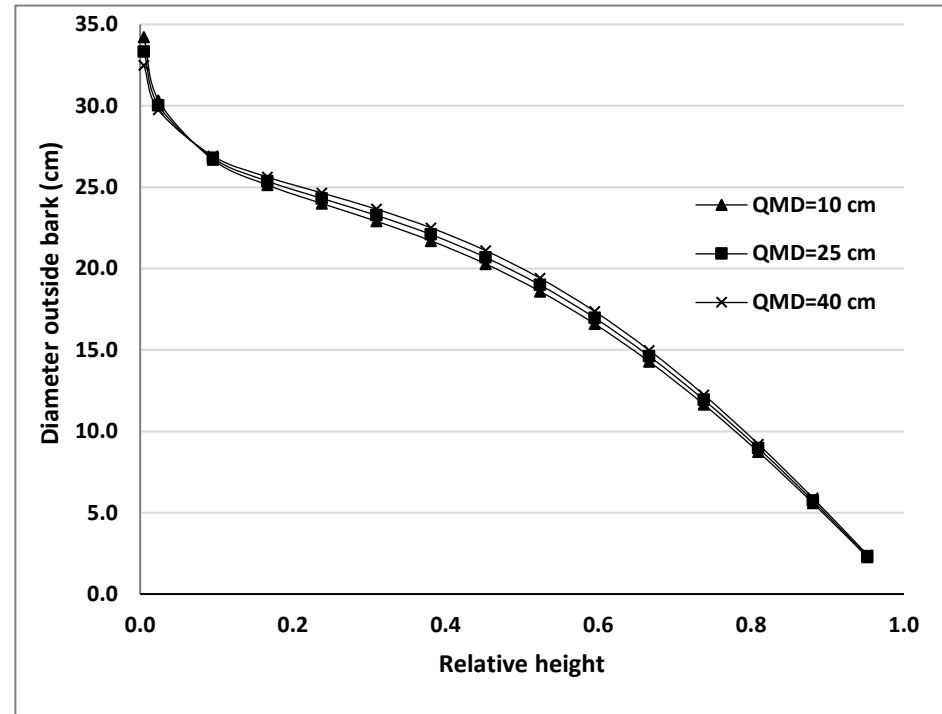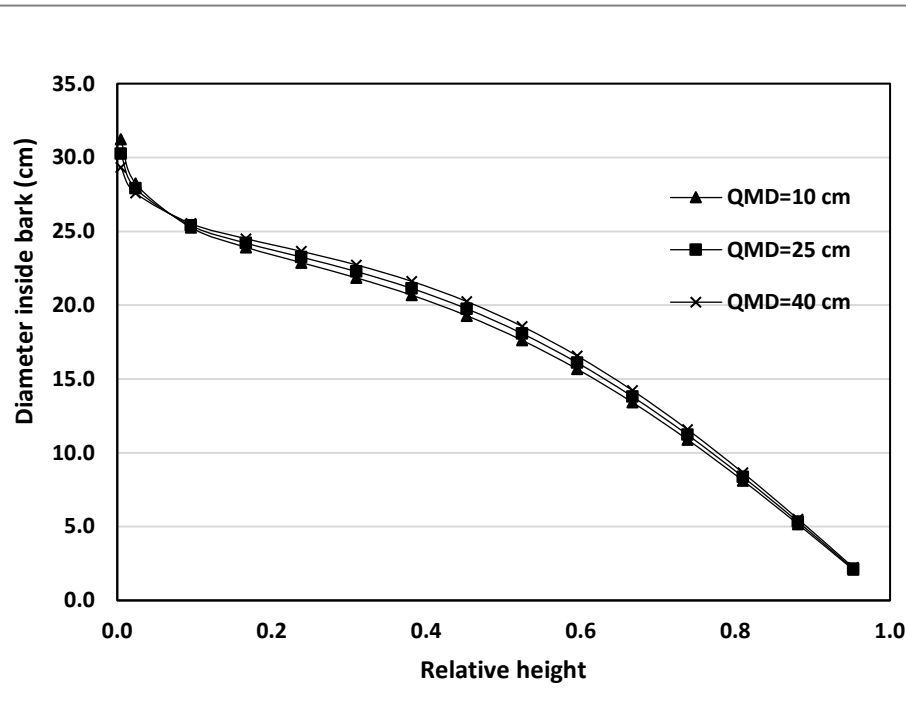Residuals (observed – predicted) of white pine plantation inside and outside bark diameters

Ontario

Taper equations

Tree profiles (mean responses) for plantation white spruce generated using DBH = 26.0 cm and total height = 21.5 m for different QMD values (10, 25, and 40 cm) for inside and outside bark diameters.

Ontario

Taper equations

Tree profiles (mean responses) for plantation white pine generated from Eq. (4) using DBH = 28.0 cm and total height = 21.0 m for different QMD values (10, 25, and 40 cm) for inside and outside bark diameters

Ontario

Taper equations

- Taper is affected by stand density for both species

- Stand density represented by QMD was the most important variable in explaining the variation in taper for both species

- Crown ratio explained some variation in taper without QMD in the model but was not significant in the presence of QMD for both species

- Taper equations should be developed by accounting for stand density

Ontario

Volume equations

Develop total inside and outside bark and merchantable volume equations for white spruce and white pine plantations

Ontario

Volume equations

Sharma and Oderwald (2001) developed a dimensionally compatible volume equation as

$$V = \alpha + \beta D^{\gamma} H^{3-\gamma} + \varepsilon \qquad (1)$$

where, V = total volume (inside/outside bark, m^3) of a tree

D = diameter at breast height (DBH; m)

H = total tree height (m)

α, β and γ are parameters to be estimated and ε is the error term.

Ontario

Volume equations

In mathematics, the volume of a circular base solid with base diameter D and height H is expressed as:

$$V = \beta D^2\, H \qquad\qquad (2)$$

where, $\beta = \pi/4$, $\pi/8$, $\pi/12$, and $\pi/16$ for a cylinder, paraboloid, cone, and neiloid, respectively.

If the shape of white spruce and white pine trees is one of these solids, tree/stem volume can be expressed as:

$$V = \alpha + \beta D^2\, H + \varepsilon \qquad (3)$$

Ontario

Volume equations

white spruce

Total inside and outside bark

$$V_{ij} = \alpha + (\beta + b_{1i})D_{ij}^2 H_{ij} + \varepsilon_{ij} \qquad (4)$$

Merchantable

$$V_{ij} = \alpha + (\beta + b_{1i})D_{ij}^\gamma H_{ij}^{3-\gamma} + \varepsilon_{ij} \qquad (5)$$

white pine

Total inside and outside bark and merchantable

$$V_{ij} = \alpha + (\beta + b_{1i})D_{ij}^\gamma H_{ij}^{3-\gamma} + \varepsilon_{ij} \qquad (6)$$

Ontario

Volume equations

Parameter estimates, standard error (Std error) and fit statistics (MSE (σ_e^2), variance of b (σ_b^2), and AIC (Akaike's information criterion)) for **inside bark** volume equations for white spruce and white spruce plantations

Parameters	White spruce		White pine	
	Estimate	Std error	Estimate	Std error
α	0.00547	0.00200	0.00412	0.00032
β	0.36070	0.00343	0.33240	0.04499
γ	--	--	1.99340	0.03340
φ *	5.00930	0.41680	4.12420	0.39670
σ_e^2	0.86240	0.53270	0.27370	0.14450
σ_b^2	0.00008	0.00005	0.00016	0.00008
AIC	-861.8	--	-741.0	--

* Weight (power of DBH (m)). Note: Before using these estimates, convert DBH to m.

Ontario

Volume equations

Parameter estimates, standard error (Std error) and fit statistics (MSE (σ_e^2), variance of b (σ_b^2), and AIC (Akaike's information criterion)) for **outside bark** volume equations for white spruce and white spruce plantations

Parameters	White spruce		White pine	
	Estimate	Std error	Estimate	Std error
α	0.00584	0.00203	0.00478	0.00031
β	0.38080	0.00350	0.35040	0.04467
γ	--	--	1.98120	0.03022
φ *	4.93510	0.42760	4.54750	0.41530
σ_e^2	0.68610	0.43400	0.50050	0.27740
σ_b^2	0.00010	0.00005	0.00017	0.00008
AIC	-879.3	--	-730.4	--

* Weight (power of DBH (m)). Note: Before using these estimates, convert DBH to m.

Ontario

Volume equations

Parameter estimates, standard error (Std error) and fit statistics (MSE (σ_e^2), variance of b (σ_b^2), and AIC (Akaike's information criterion)) for **merchantable** volume equations for white spruce and white spruce plantations

| Parameters	White spruce		White pine	
	Estimate	Std error	Estimate	Std error
α	-0.01646	0.00307	-0.00484	0.00033
β	0.12260	0.02317	0.31640	0.04549
γ	1.75580	0.04447	1.99040	0.03428
ϕ *	4.40590	0.36370	3.89640	0.39830
σ_e^2	0.31320	0.16890	0.20340	0.10670
σ_b^2	0.00001	0.00001	0.00014	0.00007
AIC	-883.5	--	-740.2	--

* Weight (power of DBH (m)).

Ontario
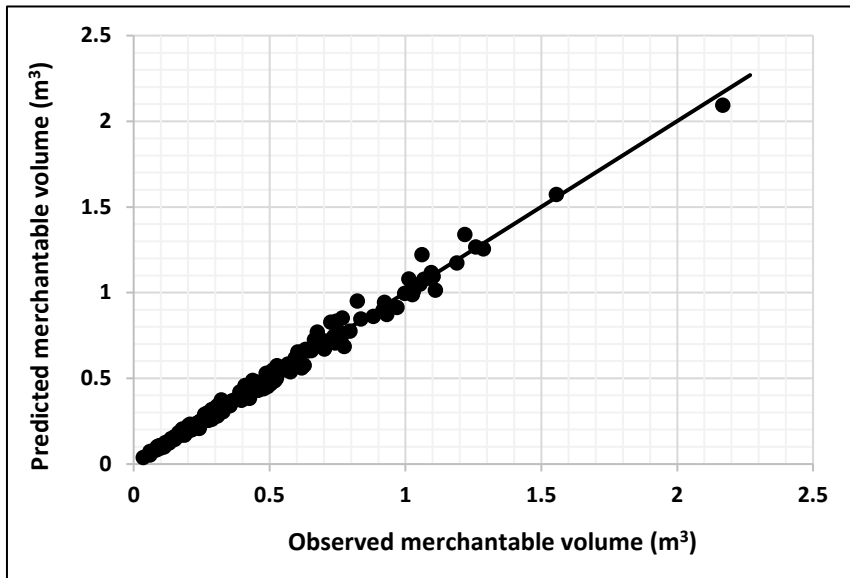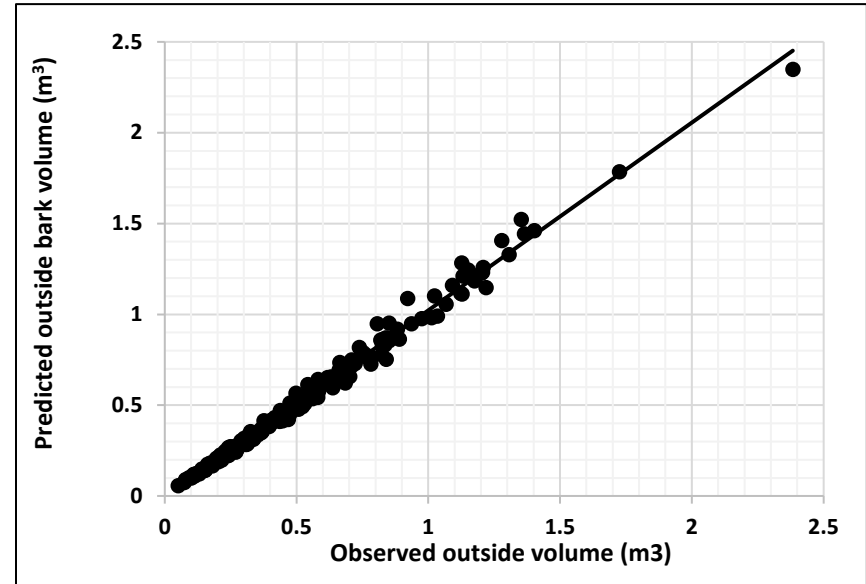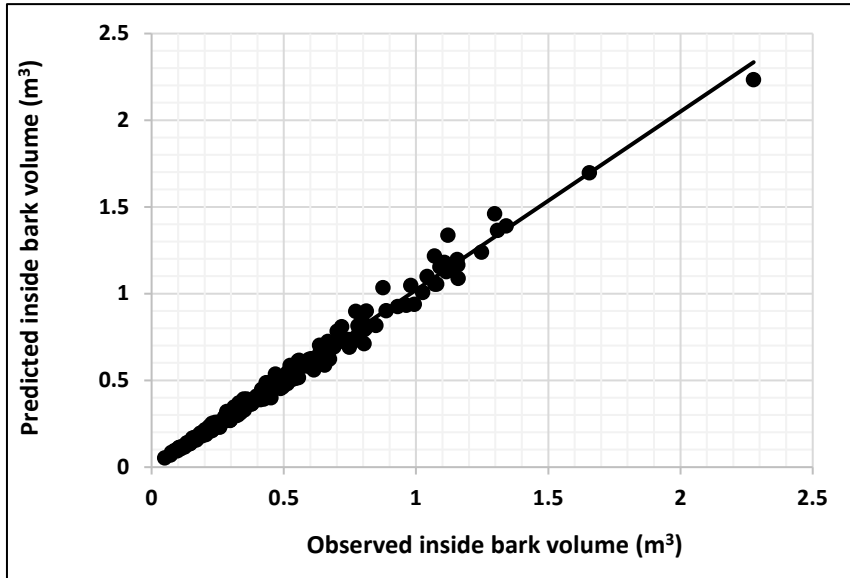
Volume equations

Predicted volumes plotted against observed volumes for white spruce plantations

Ontario
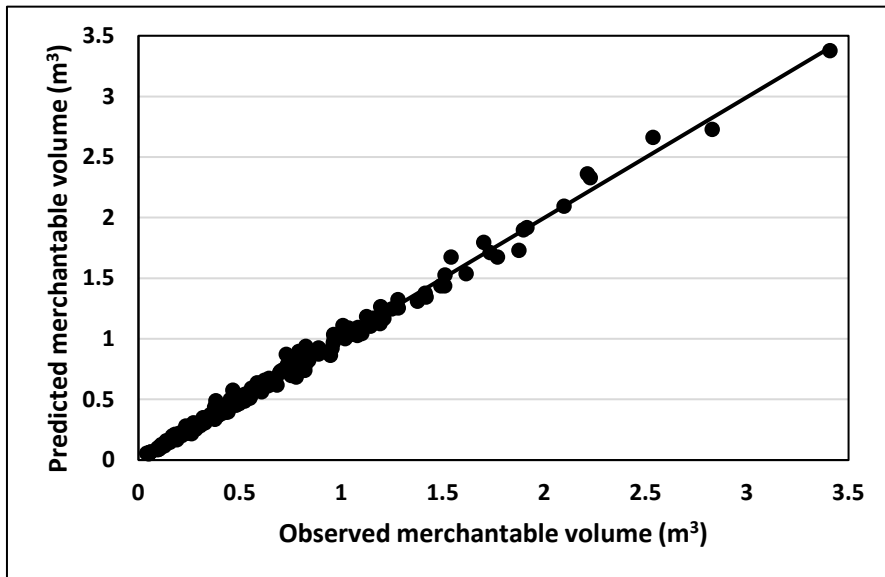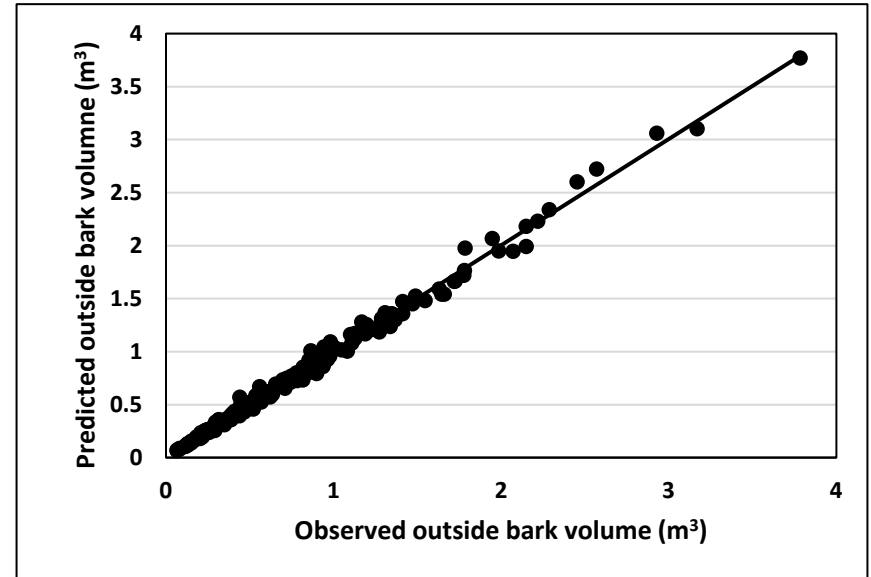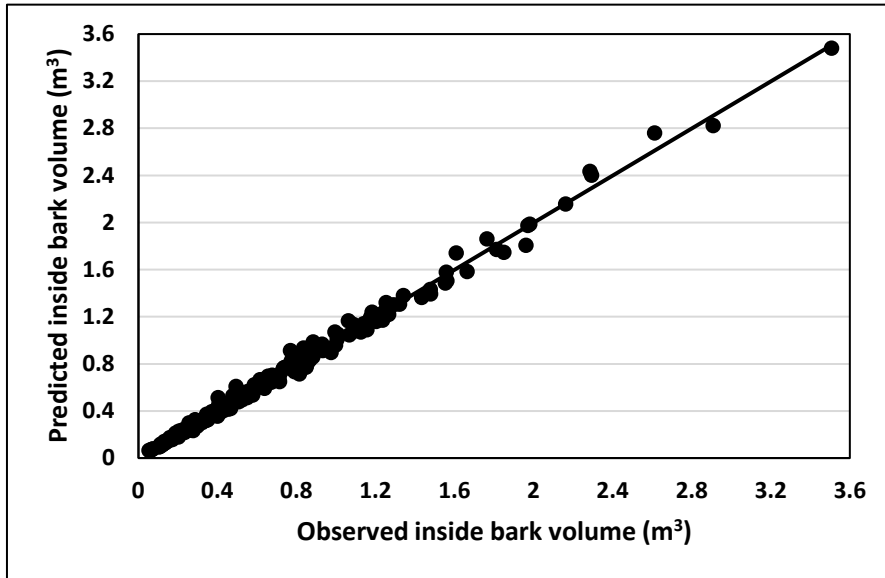
Volume equations

Predicted volumes plotted against observed volumes for white pine plantations

Ontario

Volume equations

- None of circular based solids (cylinder, paraboloid, cone, and neiloid) represented the shape of white pine and white spruce tree grown in plantations

- Model forms for total and merchantable volumes were different for white spruce but the same for white pine

Ontario

Diameter growth equations

Analyze climate effects on diameter growth of white spruce and white pine plantations and develop diameter growth equations by incorporating climate variables
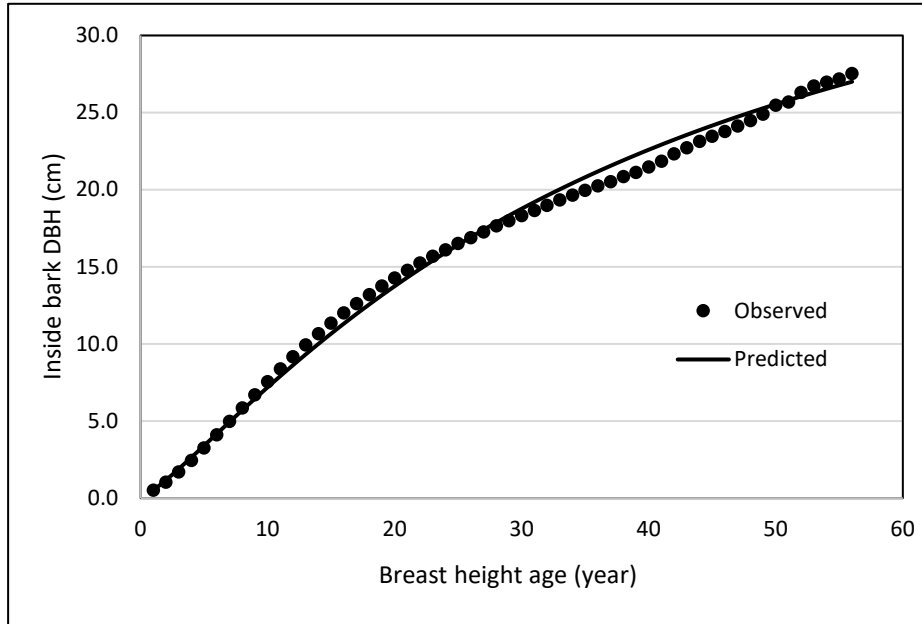
Ontario

Diameter growth equations

McDill-Amateis growth function

$$H = \frac{\beta_0}{1-\left(1-\frac{\beta_0}{H_0}\right)\left(\frac{A_0}{A}\right)^{\beta_1}} + \varepsilon \quad \Rightarrow \quad D = \frac{\beta_0}{1-\left(1-\frac{\beta_0}{D_0}\right)\left(\frac{A_0}{A}\right)^{\beta_1}} + \varepsilon \quad (1)$$

Parameters	White spruce		White pine	
	Estimates	SE	Estimates	SE
β_0	43.9032	0.36930	51.8574	0.64230
β_1	1.2177	0.00581	1.0898	0.00613
σ^2	0.18866	0.00054	0.05236	0.00075
AIC	-4309	--	-1073	--

Ontario

Diameter growth equations

White Spruce

White pine

Ontario

Diameter growth equations

Diameter growth models that include climate variables

White spruce

$$D_{ij} = \frac{\beta_0 + \beta_2 ACMI}{1 - \left(1 - \frac{\beta_0 + \beta_2 ACMI}{D_{ik(k \neq j)}}\right)\left(\frac{A_0}{A}\right)^{\beta_1 + \beta_3 GSMT + \beta_4 GSTP + \beta_5 JCMI}} + \varepsilon_{ij} \tag{2}$$

White pine

$$D_{ij} = \frac{\beta_0 + \beta_2 GMCMI}{1 - \left(1 - \frac{\beta_0 + \beta_2 GMCMI}{D_{ik(k \neq j)}}\right)\left(\frac{A_0}{A}\right)^{\beta_1 + \beta_3 GSMT + \beta_4 GSTP + \beta_5 JCMI + \beta_6 JCMI^2}} + \varepsilon_{ij} \tag{3}$$

JCMI = June CMI (climatic moisture index)

(CMI = mean monthly precipitation (MMP) – potential evapotranspiration (PET))

ACMI = Annual (Jan – Dec) sum of monthly climatic moisture indices

GMCMI = Sum of April to August moisture indices
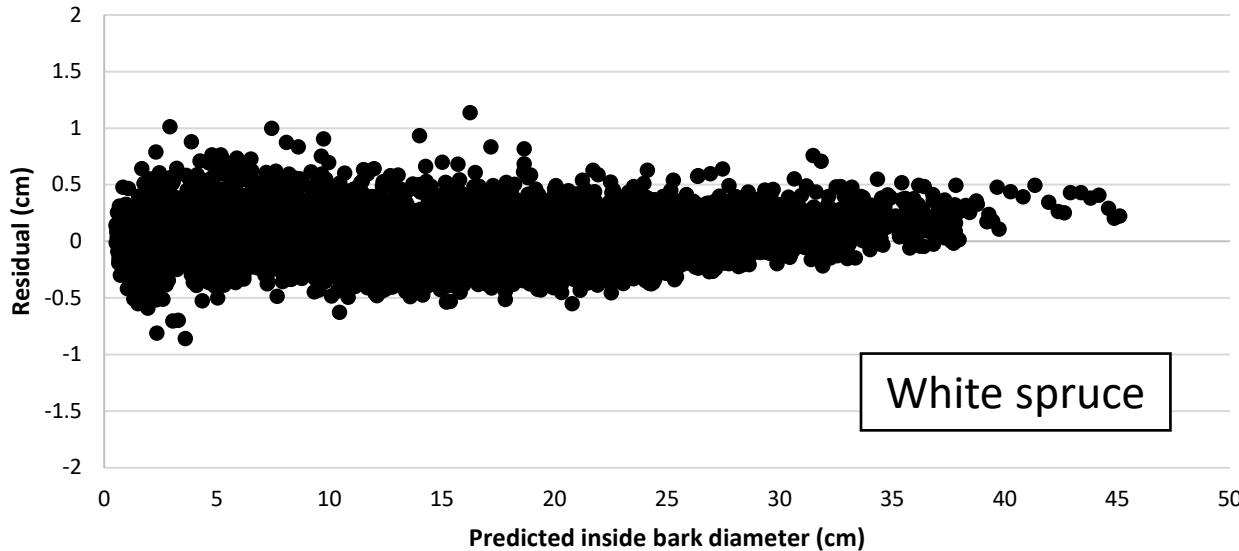
GSMT = growing season mean temperature

GSTP = growing season total precipitation

Ontario

Estimated parameters for diameter growth equations

$$D = \frac{\beta_0 + \beta_2 GMCMI}{1 - \left(1 - \frac{\beta_0 + \beta_2 GMCMI}{D}\right)\left(\frac{A_0}{A}\right)^{\beta_1 + \beta_3 GSMT + \beta_4 GSTP + \beta_5 JCMI + \beta_6 JCMI^2}} \quad \text{for white pine}$$

Parameters	White spruce		White pine	
	Estimates	SE	Estimates	SE
β_0	49.77260	0. 93160	52.9072	0.71300
β_1	1.52470	0.06365	1.36010	0.06869
β_2	-0.15850	0.02286	0.39750	0.06065
β_3	-0.03690	0.004823	-0.01308	0.00482
β_4	0.000381	0.000054	-0.00015	0.00004
β_5	0.005878	0.001331	0.00687	0.00133
β_6	--	--	-0.00071	0.00025
σ^2	0.03596	0.000531	0.05161	0.00074
AIC	-4464	--	-1203	--

Ontario

Diameter growth equations

Residuals (observed – predicted) of inside bark diameter at breast height (DBH) estimated using Eq. (2) for white spruce and Eq. (3) for white pine plotted against predicted inside bark diameters.

Ontario
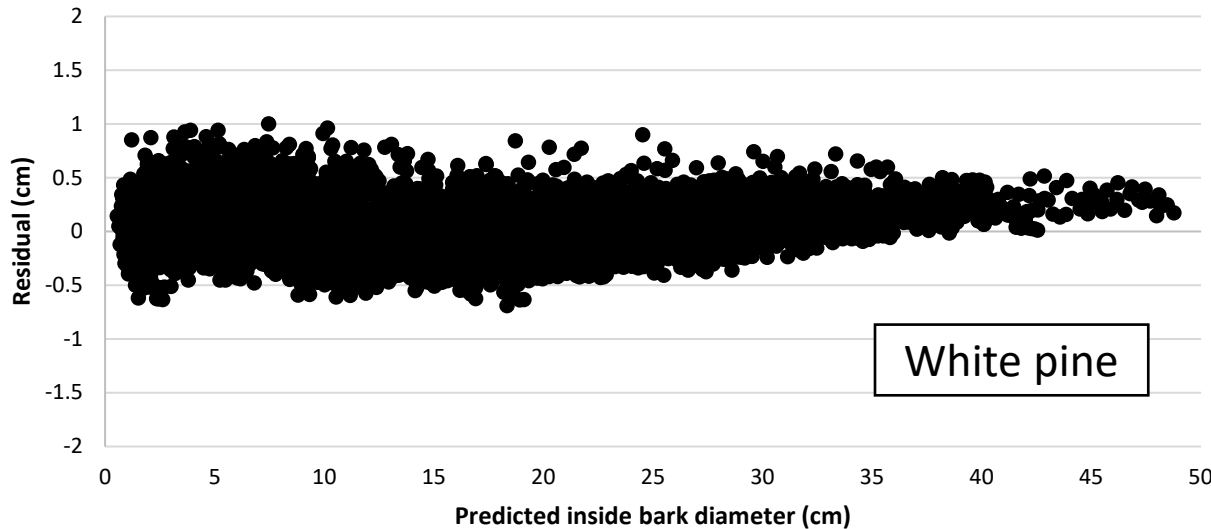
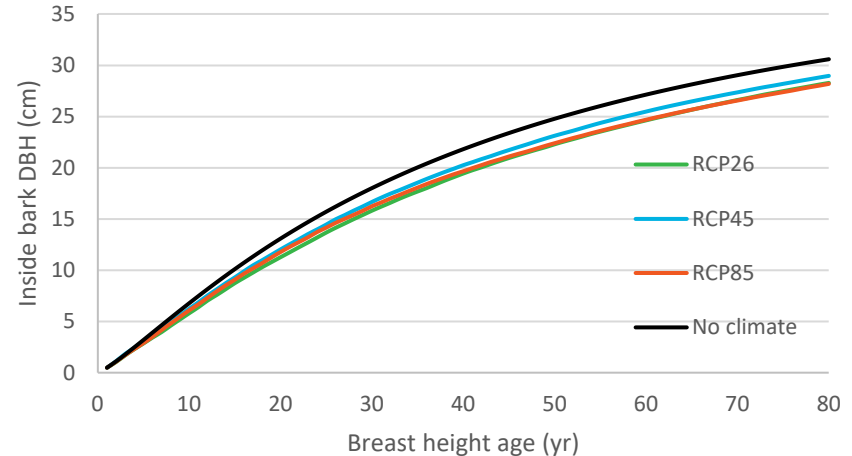# Climate effects on diameter growth – White spruce

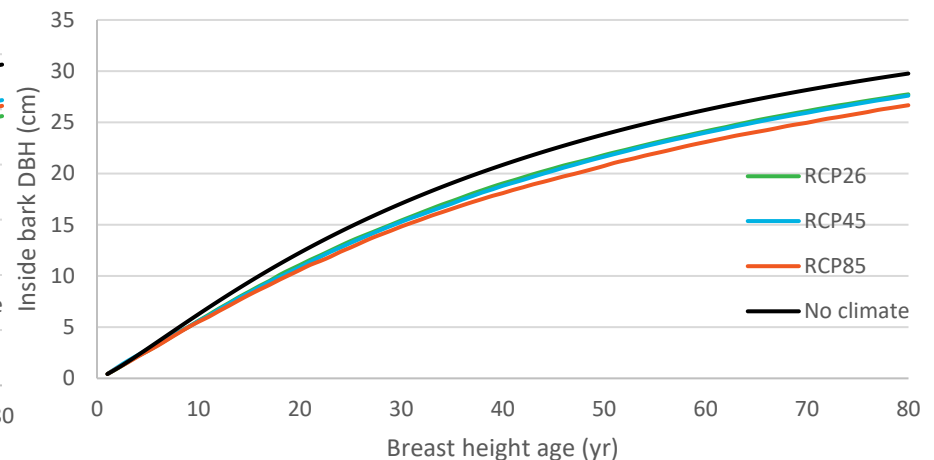| Climate Variables |
|---|
| JCMI = June CMI |
| ACMI = sum of Jan – Dec monthly CMIs |
| GSTP = growing season total precipitation |
| GSMT = growing season mean temperature |

East

West

South

Ontario
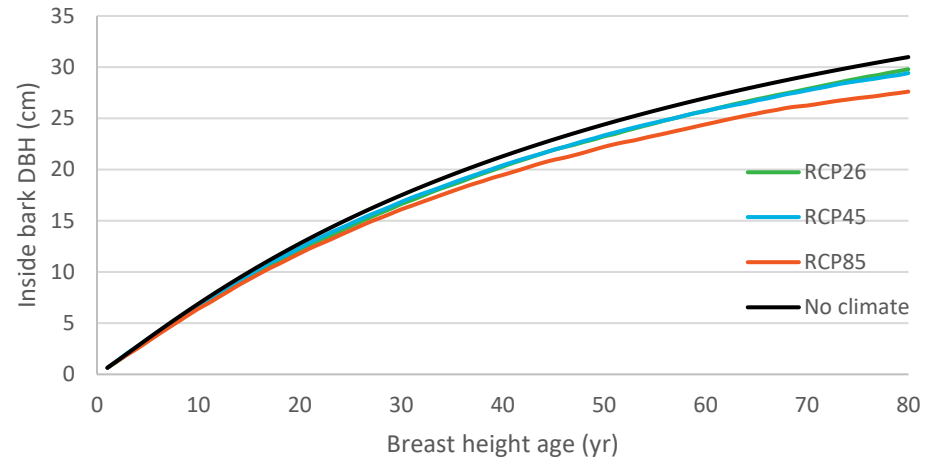
Climate effects on diameter growth – White pine
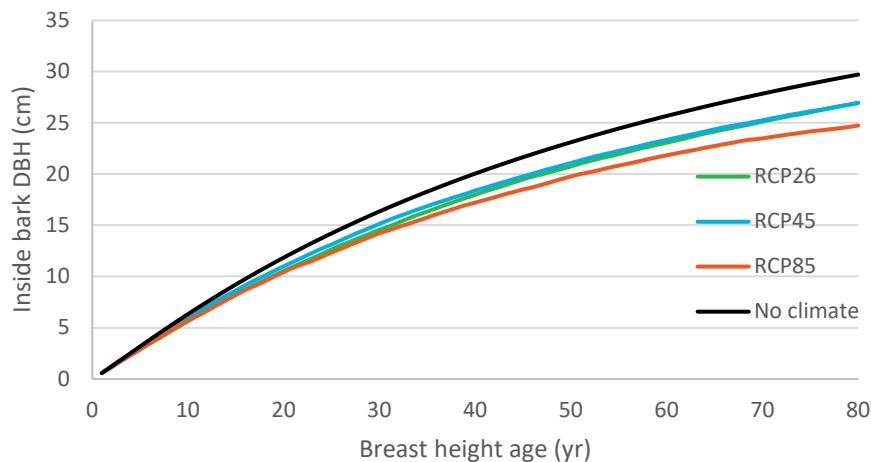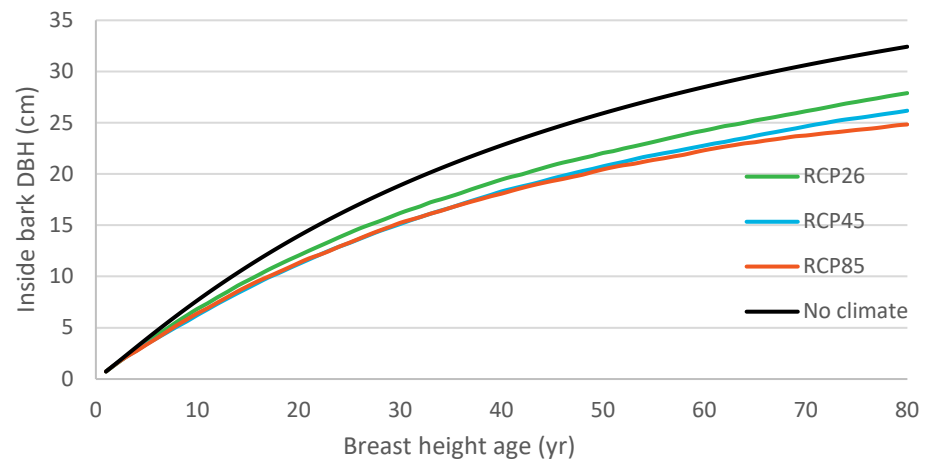
Diameter growth equations

- Diameter growth is affected by climate for both white pine and white spruce plantations

- The overall climate effect is negative for both species

- The magnitude of the effect depends on tree species and locations

- Not all climate variables affecting diameter growth are the same for both tree species

Ontario

Acknowledgements

- Forest Futures Trust — Knowledge Transfer and Tool Development (KTTD) program

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- Chris Stratton, Biodiversity and Monitoring Section, MNRF

- Todd Little, Biodiversity and Monitoring Section, MNRF

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Ontario